ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)
Anguchettypalayam, Panruti – 607106.

QUESTION BANK (R-2017)

MA 8251
ENGINEERING MATHEMATICS-II

QUESTION BANK

PERIOD: DEC-20 – MAR-21 **BATCH**: 2020 – 2024

BRANCH: MECH **YEAR/SEM:** I/02

SUB CODE/NAME: MA8251 – ENGINEERING MATHEMATICS-II

UNIT-I (MATRICES)

PART-A

1. State Cayley- Hamilton theorem.
2. Find the sum and product of the Eigenvalues of the matrix $A = \begin{pmatrix} 3 & 8 & 6 \\ 8 & 4 & 2 \\ 6 & 2 & 5 \end{pmatrix}$
3. Find the sum and product of the Eigenvalues of the matrix $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$
4. The Eigen value of a matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0, what is the third Eigen value?

   And find the product of the Eigen value?
5. Find the sum and product of all the Eigenvalues of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
6. If 2 and 3 are the two eigenvalues of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$ then find the value of b.
7. The product of two Eigenvalues of the $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16.find the third Eigenvalue.
8. Find the Eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.
9. Find the Eigenvalues of 3A+2I, where A= $\begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix}$.
10. If λ is an Eigen value of a matrix A, then λ^{-1} is the Eigen value of A^{-1}.
11. If λ is an Eigen value of a matrix A, then λ^2 is the Eigen value of A^2.
12. Prove that the Eigen value of a orthogonal matrix are of unit modulus.
13. If the Eigen value of the matrix 3x3 are 2,3,1 then find the Eigen value of adjoint of A.

14. If 2,-1,-3 are the Eigen value of the matrix A,then find the Eigen value of $A^2 - 2I$.

15. If the sum of two Eigen values and trace of a 3x3 matrix A are equal, find the value of |A|.

16. Prove that $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2zx = 0$ is indefinite.

17. Give the nature of a quadratic from whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

18. What is the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables?

19. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

20. Write down the matrix corresponding to the quadratic form $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$.

PART-B

CHAPTER-1.1 **(8-MARKS)**

1. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
2. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$
3. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$
4. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
5. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
6. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$
7. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$
8. Find the **Eigen values** and **Eigen vectors** for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

CHAPTER-1.2 (8-MARKS)

1. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{-1}}$ for the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

2. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{-1}}$ for the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \\ 2 & 1 & -2 \end{bmatrix}$

3. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{-1}}$ for the matrix $\begin{bmatrix} -3 & 2 & 1 \\ 3 & -1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

4. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{-1}}$ for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

5. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{-1}}$ for the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$

6. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{4}}$ for the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

7. **Use Cayley-Hamilton theorem to find the value of**

   $\boldsymbol{A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I}$ Where A=$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

8. Verify the **Cayley-Hamilton** theorem and also find $\boldsymbol{A^{4}}$ for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

CHAPTER-1.3 (16-MARKS)

1. Reduce the quadratic form into the canonical by using orthogonal transform $6\boldsymbol{x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1}$ and also find Rank, signature, Index

2. Reduce the quadratic form into the canonical by using orthogonal transform $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ and also find Rank, signature, Index

3. Reduce the quadratic form into the canonical by using orthogonal transform $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ and also discuss the nature

4. Reduce the quadratic form into the canonical by using orthogonal transform $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_2x_3 + 2x_3x_1$ and also discuss the nature

5. Reduce the quadratic form into the canonical by using orthogonal transform $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ and also find Rank, signature, Index

6. Reduce the quadratic form into the canonical by using orthogonal transform $3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz$. and also discuss the nature

7. Reduce the quadratic form into the canonical by using orthogonal transform $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. and also find Rank, signature, Index

8. Reduce the quadratic form into the canonical by using orthogonal transform $x_1^2 + 2x_2^2 + x_3^2 - 12x_1x_2 + 2x_2x_3$ and also find Rank, signature, Index

9. Reduce the quadratic form into the canonical by using orthogonal transform $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ and also find Rank, signature, Index

ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: DEC-20 – MAR-21 **BATCH**: 2020 – 2024

BRANCH: MECH **YEAR/SEM:** I/02

SUB CODE/NAME: MA8251 – ENGINEERING MATHEMATICS-II

UNIT-II (VECTOR CALCULUS)

PART-A

1. Find $\boldsymbol{\lambda}$ such that $\vec{F} = (3x - 2y + z)\vec{i} + (4x + \lambda y - z)\vec{j} + (x - y + 2z)\vec{k}$ is Solendial
2. Find $\boldsymbol{\lambda}$ such that $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + 2\lambda z)\vec{k}$ is Solendial
3. Find the values of **a,b,c** where $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ is irrotational.
4. Find the unit normal vector to the surface $x^2y + 2xz = 4$ at (2,-2,3)
5. Find the unit normal vector $xy = z^2$ *, at the point* $(1,1,-1)$
6. Find the unit normal vector to the surface $x^2 + xy + z^2 = 4$ at the point (1,-1,2)
7. Find the directional derivative of the function $x^2 + 2xy$ at (1,-1,3) in the direction $\vec{i} + 2\vec{j} + 2\vec{k}$
8. Find the directional derivative of the function $x^2yz + 4xz^2 + xyz$ at (1,2,3) in the direction $2\vec{i} + \vec{j} - \vec{k}$
9. Find the directional derivative of the function $\emptyset = xy^2z^3z$ at (1,1,1) along the normal to the surface $x^2 + xy + z^2 = 3$ at the point (1,1,1)
10. Using Green's theorem evaluate $\int(xdy - y\,dx)$, where C is the circle $x^2 + y^2 = 1$ in the xy plan
11. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ such that $|\vec{r}| = r$, prove that

(i) $\nabla r^n = nr^{n-2}\vec{r}$

(ii) $\nabla f(r).\vec{r} = 0$

(iii) Gard $\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$

PART-B

I- GREEN'S THEOREM

1. Verify Green's theorem in the XY-plane for $\int_c (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region given by $x = 0, y = 0, x + y = 1$.

2. Verify Green's theorem for $\int_c (x^2 + y^2)\,dx - 2xy\,dy$, where C is taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

3. Verify Green's theorem in the plan for $\int_c (3x - 8y^2)\,dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = y^2, y = x^2$.

4. Verify Green's theorem in the plan for $\int_c (x^2 - xy^3)\,dx + (y^2 - 2xy)dy$, where C is the square with vertices $(0,0), (2,0), (2,2), (0,2)$ (or) $x = 0, x = 2, y = 0, y = 2$

5. Verify Green's theorem in the plan for $\int x^2 dx + xy\,dy$, where C is the curve in the XY plane given by $x = 0, y = 0, x = a, y = a$.

6. Verify Green's theorem in the XY-plane for $\int_c (xy + y^2)\,dx + x^2 dy$,where C is the closed curve of the region bounded by $y = x \text{ and } y = x^2$.

7. Apply the Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\int(xdy - y\,dx)$. Hence find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum

8. Using Green's theorem, Evaluate , $\int_c (y - \sin x)\,dx + \cos x dy$, where C is the triangle OAB where $O = (0,0), A = \left(\frac{\pi}{2}, 0\right), A = \left(\frac{\pi}{2}, 1\right)$

9. Evaluate by Green's theorem $\int_c (xy + y^2)\,dx + (x^2 + y^2)dy$ where C is the square formed by $x = -1, x = 1, y = -1, y = 1$

II- GAUSS DIVERGENCE THEOREM

1. Verify the Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$. **(OR)** $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$
2. Verify the Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$

   taken over the rectangular parallelopiped $x = 0, x = a, y = 0, y = b, z = 0, z = c$. **(OR)** $0 \le x \le a, 0 \le x \le b, 0 \le x \le c$. **(OR)** $x = 0, x = 1, y = 0, y = 2, z = 0, z = 3$
3. Verify the Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the cube bounded by the lines $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
4. Verify the Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the lines $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$
5. Verify the Gauss divergence theorem for $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ taken over the cube bounded by the lines $x = 0, x = a, y = 0, y = a, z = 0, z = a$
6. Verify the Gauss divergence theorem for $\vec{F} = xy^2\vec{i} + yz^2\vec{j} + zx^2\vec{k}$ over the region bounded by the lines $x = 0, x = 1, y = 0, y = 2, z = 0, z = 3$
7. Verify the Gauss divergence theorem for the function $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0$ *and* $z = 2$.
8. Evaluate $\iint_s \vec{F}\,\hat{n}\,ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ **and** S is the surface bounded by the region $x^2 + y^2 = 4, z = 0$ *and* $z = 3$ by using divergence theorem.
9. Use divergence theorem to evaluate $\iint_s \vec{F}\,\hat{n}\,ds$ where $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

III- STOKE'S THEOREM

1. Verify Stokes's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$, taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

2. Verify Stokes's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$, taken around the rectangle bounded by the lines $x = 0, x = a, y = 0, y = b$.

3. Verify Stokes's theorem for $\vec{F} = (y - z)\vec{i} + yz\vec{j} - xz\vec{k}$, where S is the surface

   bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$

(or) $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$ above the XOY plane.

4. Verify Stokes's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$, over the open surface of the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ not included in the
5. Using Stokes theorem Evaluate $\int \vec{F}.\overrightarrow{dr}$, where $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x + z)\vec{k}$ and C is the boundary of the triangle with vertices at $(0, 0, 0), (1, 0, 0), (1, 1, 0)$
6. Verify Stokes's theorem for $\vec{F} = x^2\vec{i} + xy\vec{j}$, taken around the rectangle bounded by the lines $x = 0, x = a, y = 0, y = a$.
7. Verify Stokes's theorem for $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ and for the surface S of the upper hemisphere $x^2 + y^2 + z^2 = 1$ and C is the circle $x^2 + y^2 = 1, z = 0$.
8. Verify Stokes's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.
9. Verify Stokes's theorem for $\vec{F} = y^2\vec{i} + xy\vec{j} - xz\vec{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$.

IV- Irrotational and Solenoidal

1. Prove that $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ is irrotational and hence find its scalar **potential.**
2. Prove that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential.
3. Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find its scalar potential function $\emptyset$ Such that $\vec{F} = \nabla\emptyset$.
4. Prove that $\vec{F} = (2xy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - 2zx)\vec{k}$ is irrotational and find its scalar potential function $\emptyset$ Such that $\vec{F} = \nabla\emptyset$.
5. Prove that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2xz^2 - y + 2x)\vec{k}$ is irrotational and find its scalar potential function $\emptyset$ Such that $\vec{F} = \nabla\emptyset$.
6. Prove that $\vec{F} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$ is irrotational and find its scalar potential.
7. Prove that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is irrotational and find its scalar Potential function $\emptyset$.

ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: DEC-20 – MAR-21 **BATCH**: 2020 – 2024

BRANCH: MECH **YEAR/SEM:** I/02

SUB CODE/NAME: MA8251 – ENGINEERING MATHEMATICS-II

UNIT-III LAPLACE TRANSFORMS

PART-A

1. State sufficient conditions for the existence of Laplace transform.
2. Define Laplace transform.
3. State initial and final value theorem.
4. Verify intial value theorem for $1 + e^{-2t}$
5. Find the $L\left[\frac{\sin t}{t}\right]$
6. Find the $L\left[e^{-t}\sin 2t\right]$
7. Find the $L\left[t\cos t\right]$
8. Find the $L\left[\frac{t}{e^{t}}\right]$
9. Find $L^{-1}\left[\frac{1}{(s^2+6s+13)}\right]$
10. Find $L^{-1}\left[\frac{1}{(s^2+4s+4)}\right]$

PART-B

I-PERIODIC FUNCTIONS

1. Find the Laplace transform of the Half-sine wave rectifier function

   i) $f(t) = \begin{cases} \sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0 & , \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ and $f\left(t + \frac{2\pi}{w}\right) = f(t)$

   ii) $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0 & , \pi < t < 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$

2. Find the Laplace transform of the triangular wave function

   i) $f(t) = \begin{cases} t & , 0 < t < 1 \\ 2 - t, & 1 < t < 2 \end{cases}$ and $f(t + 2) = f(t)$

   ii) $f(t) = \begin{cases} t & , 0 < t < \pi \\ 2\pi - t, & \pi < t < 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$

3. Find the Laplace transform of the rectangular wave function

   $f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$

4. Find the Laplace transform of the square wave function

   i) $f(t) = \begin{cases} A & , 0 < t < \frac{T}{2} \\ -A & , \frac{T}{2} < t < T \end{cases}$ and $f(t + 2T) = f(t)$

   ii) $f(t) = \begin{cases} 1 & , 0 < t < \frac{a}{2} \\ -1 & , \frac{a}{2} < t < a \end{cases}$ and $f(t + a) = f(t)$

II- Convolution Theorem

1. Using convolution theorem, find i) $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ **(ii)** $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$

2. Using convolution theorem, find **i)** $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$ **(ii)** $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$

3. Using convolution theorem, find **i)** $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$ **(ii)** $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$

4. Using convolution theorem, find i) $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ **ii)** $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$

5. Using convolution theorem, find $L^{-1}\left[\frac{2}{(S+1)(s^2+4)}\right]$

6. Using convolution theorem, find $L^{-1}\left[\frac{s^2+s}{(s^2+1)\,(s^2+2s+2)}\right]$

III- Solve the differential equations using Laplace transform

1. Using Laplace transform ,solve $y'' - 2y' + y = e^t,\ where\ \ y(0) = 2, y'(0) = 1.$
2. Solve the differential equation , using Laplace

$$y'' + 6y' + 5y = e^{-2t},\ \ given\ that \quad y(0) = 0, y'(0) = 1.$$

3. Solve the differential equation , using Laplace

$$y'' + 4y' + 4y = e^{-t},\ \ given\ that \quad y(0) = 0, y'(0) = 0.$$

4. Solve by using Laplace $y'' - 3y' + 2y = 4,\ \ given\ that \quad y(0) = 2, y'(0) = 3.$
5. Solve by using Laplace $(D^2 + 4)y = \sin 2t,\ given\ that\ \ y = 3, \frac{dy}{dt} = 4\ \ at\ \ t = 0$
6. Solve by using Laplace $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t\ ,\ where\ \ y = 0, \frac{dy}{dt} = 0\ \ at\ \ t = 0$
7. Solve by using Laplace $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t\ ,\ where\ \ y(0) = 0, y'(0) = 1$

8. Solve by using Laplace $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 1$, $where\ y(0) = 0, y'(0) = 1$

IV- Using Laplace transform

1. Find the $L\left[\frac{\cos at - \cos bt}{t}\right]$
2. Find the $L\left[\frac{e^{-at} - e^{-bt}}{t}\right]$
3. Find the $L\left[\frac{1-\cos t}{t}\right]$
4. Find the $L\,[t^2\, e^{-2t} \cos t]$
5. Find the $L\left[\int_0^t e^{-t}\, t \sin t\, dt\right]$
6. Find the $\left[e^{-t} \int_0^t t \cos t\, dt\right]$
7. Find the $L\left[e^{2t} \int_0^t \frac{\sin 3t}{t}\, dt\right]$
8. Find the $L\left[\int_0^t \frac{e^t \sin t}{t}\, dt\right]$
9. Evaluate using Laplace transform $\int_0^\infty e^{-2t}\, t \sin 3t\, dt$
10. Evaluate using Laplace transform $\int_0^\infty \frac{e^{-2t} \sin^2 t}{t}\, dt$

ST.ANNE'S
COLLEGE OF ENGINEERING AND TECHNOLOGY
(An ISO 9001:2015 Certified Institution)
Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: DEC-20 – MAR-21 **BATCH**: 2020 – 2024

BRANCH: MECH **YEAR/SEM:** I/02

SUB CODE/NAME: MA8251 – ENGINEERING MATHEMATICS-II

UNIT-IV (ANALYTIC FUNCTIONS)

PART-A

1. If $f(z) = z^2$ analytic? Justify
2. Show that $f(z) = |z|^2$ is differentiable at $z = 0$ but not analytic at $z = 0$.
3. Check whether $w = \bar{z}$ is analytic everywhere or not.
4. For what values of a,b,c the function $f(z) = (x - 2ay) + i(bx - cy)$ is analytic.
5. Find the constants a,b if $f(z) = x + 2ay + i(3x + by)$ is analytic.
6. Find the constants a,b,c if $f(z) = x + ay + i(bx + cy)$ is analytic.
7. Prove that $w = \sin 2z$ is analytic function.
8. Find the fixed point (or) Invariant point (or) critical point

   i) $w = \frac{1+z}{1-z}$ **ii)** $w = \frac{z-1}{z+1}$ **iii)** $w = \frac{6z-9}{z}$ **iv)** $w = 1 + \frac{2}{z}$ **v)** $f(z) = z^2$
9. Prove that the bilinear transformation has atmost two fixed point.
10. Show that $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is a harmonic.
11. Prove that $u = e^x \sin y$ is harmonic.
12. Find the value of 'm' if $u = 2x^2 - my^2 + 3x$ is harmonic.
13. Find the image of the circle $|z| = 3$ under the transformation $w = 2z$.

PART-B

I- Bilinear Transformation

1. Find the bilinear transformation of the points $Z = 1, i, -1$ $into$ $W = 0, 1, \infty$

2. Find the bilinear transformation of the points $Z = -1, -i, 1$ $into$ $W = \infty, i, 0$

3. Find the bilinear transformation of the points $Z = 0, 1, \infty$ $into$ $W = i, 1, -i$

4. Find the bilinear transformation of the points $Z = 1, i, -1$ $into$ $W = i, 0, -i$

5. Find the bilinear transformation of the points $Z = -1, 0, 1$ $into$ $W = -1, -i, 1$

6. Find the bilinear transformation of the points $Z = 1, i, -1$ $into$ $W = 2, i, -2$

7. Find the bilinear transformation of the points $Z = -i, 0, i$ $into$ $W = -1, i, 1$

8. Find the bilinear transformation of the points $Z = 0, -1, i$ $into$ $W = i, 0, \infty$

II- Conformal Mapping

1. Find the image of $|Z - 1| = 1$ under the map $w = \frac{1}{z}$.

2. Find the image of $|Z - 1| = 1$ under the map $w = \frac{1}{z}$.

3. Find the image of $|Z + i| = 1$ under the map $w = \frac{1}{z}$.

4. Find the image of $|Z - 2i| = 2$ under the map $w = \frac{1}{z}$.

5. Find the image of $1 < y < 2$ under the tansformation $w = \frac{1}{z}$

6. Find the image of the following region under the transformation $w = \frac{1}{z}$

i) the half plane $x > c$ *when* $c > 0$ ii) the half $y > c$ *when* $c < 0$

iii) the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ iv) the infinite strip $0 < y < \frac{1}{2}$

7. Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the **W- plane** into circles or straight lines in the **Z-plane.**

III- Find the analytic functions $f(z) = u + iv$

1. Find an analytic function, whose real part is given $u(x, y) = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

2. Find an analytic function, whose real part is given $u(x, y) = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$

3. Find an analytic function, whose real part is given

   $u(x, y) = e^x(x\cos y - y\sin y)$

4. Find an analytic function $f(z) = u + iv$, if $u = e^{2x}(x\cos 2y - y\sin 2y)$

5. Find an analytic function $f(z) = u + iv$, **if** $u = e^{-2xy}\sin(x^2 - y^2)$

6. Find an analytic function $f(z) = u + iv$ *if* $v = e^{-x}(x\cos y + y\sin y)$

7. If $f(z) = u + iv$ is analytic, find $f(z)$ given that $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

8. Determine the analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$

IV-Harmonic Functions

1. To prove that $\boldsymbol{u(x,y) = \frac{1}{2}\log(x^2+y^2)}$ is harmonic function and also find its conjugate harmonic.
2. Find the function $\boldsymbol{u(x,y) = x^2 - y^2}$ and $\boldsymbol{v\,(x,y) = \frac{-y}{x^2+y^2}}$ is harmonic function.
3. Prove that $\boldsymbol{e^x(x\cos y + y\sin y)}$ can be the real part of an analytic function and determine its harmonic conjugate.

V- Properties and Results

1. Prove that the real and imaginary parts of an analytic functions are harmonic.
2. If $f(z) = u(x,y) + iv(x,y)$ is an analytic function, show that the curves $u(x,y) = c_1 and\ v(x,y) = c_2$ Cut orthogonally.
3. An analytic $\boldsymbol{f(z) = u + iv}$ function with constants modulus is constants.
4. If $\boldsymbol{f(z)}$ is a regular function of Z , prove that $\boldsymbol{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2}$.

ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: DEC-20 – MAR-21 **BATCH**: 2020 – 2024

BRANCH: MECH **YEAR/SEM:** I/02

SUB CODE/NAME: MA8251 – ENGINEERING MATHEMATICS-II

UNIT-V (COMPLEX INTEGRATION)

PART-A

1. State the Cauchy's integral theorem.
2. State the Cauchy's integral formulae.
3. State the Cauchy's residue theorem.
4. Define Singular point.
5. Define Isolated singular points.
6. Define Essential singular point.
7. Define Removable singular point.
8. Find the residue of $f(z) = \frac{z^2}{(z-2)(z+1)^2}$ $at\ z = 2$
9. Find the residue of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ $at\ the\ singular\ point\ z = 1$
10. Evaluate $\int_c \frac{z}{(z-2)} dz\ , where\ C\ \ is\ the\ circle\ |z| = 1$
11. Evaluate $\int_c \frac{e^z}{z+1} dz\ , where\ C\ \ is\ the\ circle\ \left|z + \frac{1}{2}\right| = 1$
12. Evaluate $\int_c \frac{e^{2z}}{z^2+1} dz\ , where\ C\ \ is\ the\ circle\ |z| = \frac{1}{2}$
13. Find the residue of $\frac{1-e^{2z}}{z^4}$ $at\ z = 0$
14. Find the residue of $\frac{1-e^{-z}}{z^3}$ $at\ z = 0$
15. Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ $at\ the\ simple\ pole.$
16. Find the residue of $f(z) = ze^{2/z}\ at\ z = 0$
17. Find the residue of $f(z) = e^{1/z}\ at\ z = 0$
18. Find the residue of $f(z) = \frac{\sin z}{z^4}$ $at\ z = 0$

19. Find the residue of $f(z) = z\cos\frac{1}{z}$ $at\, z = 0$

20. Find the poles of the function $f(z) = \frac{1}{(z+1)(z-2)^2}$ and find the residue at the simple pole.

PART-B

I- Find the Laurent's series.

1. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the series in the regions
   $i)\ |z| < 2$ $ii)\ |z| > 3$ $iii)\ 2 < |z| < 3$ using Laurent's series

2. Expand $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ in Laurent's series if the
   $i)\ |z| < 2$ $ii)\ |z| > 3$ $iii)\ 2 < |z| < 3$ $iv)\ 1 < |z+1| < 3$

3. Expand in Laurent's series of $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$

4. Expand
   $f(z) = \frac{1}{(z-1)(z-2)}$ $i)\ |z| < 1$ $ii) |z| > 2$ $iii) 1 < |z| < 2$ $iv)\ 0 < |z-1| < 1$

5. Expand $f(z) = \frac{z}{(z-1)(z-2)}$ in the region $i)\ |z| < 1$ $ii) |z| > 2$
   $iii) 1 < |z| < 2$ $iv)\ |z-1| < 1$

6. Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in the region $i)\ |z| < 1$ $ii) |z| > 2$
   $iii) 1 < |z| < 2$ $iv)\ |z+1| < 1$

7. $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region
   $i)\ |z| < 2$ $ii) |z| > 3$ $iii) 2 < |z| < 3$ $iv)\ |z+1| < 2$

II-Cauchy's Integral Formulae

1. Evaluate $\int_c \frac{z^2+1}{z^2-1}dz$ where C is a circle of unit radius and centre at $Z = 1$.

2. Evaluate $\int_c \frac{z^2+1}{z^2-1}dz$ where C is a circle of unit radius and centre at $Z = -1$.

3. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+4}{z^2+2z+5}dz\ , where\ C\ \ is\ the\ circle\ |z+1-i| = 2$

4. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+4}{z^2+2z+5}dz\ , where\ C\ \ is\ the\ circle\ |z+1-i| = 2$

5. Using Cauchy's integral formula, evaluate

$\int_c \frac{z}{(z-1)^2(z+2)}dz\ , where\ C\ \ is\ the\ circle\ |z| = \frac{3}{2}$

6. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+1}{(z-3)(z-1)}dz\ , where\ C\ \ is\ the\ circle\ |z| = 2$

7. Using Cauchy's integral formula, evaluate

$\int_c \frac{z}{(z-2)^2(z-1)}dz\ , where\ C\ \ is\ the\ circle\ |z-2| = \frac{1}{2}$

8. Using Cauchy's integral formula, evaluate

$\int_c \frac{z+1}{z^2+2z+4}dz\ , where\ C\ \ is\ the\ circle\ |z+1+i| = 2$ **(or)** $|z+1-i| = 2$

9. Using Cauchy's integral formula, evaluate

$\int_c \frac{\cos \pi z^2+\sin \pi z^2}{(z-1)(z-2)}dz\ , where\ C\ \ is\ the\ circle\ |z| = 3$

10. If $\int_c \frac{3z^2+7z+1}{z-a}dz\ ,\ where\ c\ is\ |z| = 2,$ find $f(3),\ f(1-i), f'(1-i)$

III-Cauchy’s Residue Theorem

1. Evaluate $\int_c \frac{\cos \pi z^2+\sin \pi z^2}{(z-1)(z-2)} dz$, $where\ C\ \ is\ the\ circle\ |z| = 3$ using Cauchy’s residue theorem.
2. Evaluate $\int_c \frac{\cos \pi z^2+\sin \pi z^2}{(z-1)^2(z-2)} dz$, $where\ C\ \ is\ the\ circle\ |z| = 3$ using Cauchy’s residue theorem.
3. Evaluate $\int_c \frac{\cos \pi z^2+\sin \pi z^2}{(z-1)(z-2)^2} dz$, $where\ C\ \ is\ the\ circle\ |z| = 3$ using Cauchy’s residue theorem.
4. Evaluate $\int_c \frac{(z-1)}{(z-1)^2(z-2)} dz$, $where\ C\ \ is\ the\ circle\ |z-i| = 2$ using Cauchy’s residue theorem.
5. Evaluate $\int_c \frac{(z-1)}{(z+1)^2(z-2)} dz$, $where\ C\ \ is\ the\ circle\ |z-i| = 2$ using Cauchy’s residue theorem.
6. Evaluate $\int_c \frac{z}{(z^2+1)^2} dz$, $where\ C\ \ is\ the\ circle\ |z-i| = 1$ using Cauchy’s residue theorem.

IV-Contour Integration

TYPE-I

1. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta \ \ d\theta}{5+4\cos\theta}$ using contour integration.
2. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta \ \ d\theta}{5-4\cos\theta}$ using contour integration.
3. Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$ using contour integration.
4. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ using contour integration

TYPE-II

1. Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$.
2. Evaluate by contour integration $\int_{0}^{\infty} \frac{dx}{(x^2+a^2)^2}$
3. Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{a+b}$, $a > b > 0$ using contour integration.
4. Using contour integration ,evaluate $\int_{0}^{\infty} \frac{x^2 dx}{(x^2+1)^2}$ $if\ a > 0.$
5. Using contour integration ,evaluate $\int_{0}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.